

# The quantum geometric limit

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## Abstract

This letter analyzes the limits that quantum mechanics imposes on the accuracy to which spacetime geometry can be measured. By applying the fundamental physical bounds to measurement accuracy to ensembles of clocks and signals moving in curved spacetime – e.g., the global positioning system – I derive a covariant version of the quantum geometric limit: the total number of ticks of clocks and clicks of detectors that can be contained in a four volume of spacetime of radius  $r$  and temporal extent  $t$  is less than or equal to  $rt/\pi\ell_P t_P$ , where  $\ell_P$ ,  $t_P$  are the Planck length and time. The quantum geometric limit bounds the number of events or ‘ops’ that can take place in a four-volume of spacetime: each event is associated with a Planck-scale area. Conversely, I show that if each quantum event is associated with such an area, then Einstein’s equations must hold. The quantum geometric limit is consistent with and complementary to the holographic bound which limits the number of bits that can exist within a spatial three-volume.

Recent advances on the physical bounds to measurement accuracy have allowed the derivation of a quantum geometric limit, an intrinsically quantum mechanical limit to the accuracy with which the geometry of spacetime can be measured [1-2]. This letter derives an explicitly covariant version of this limit. The derivation connects the quantum geometric limit to previous limits to measuring spacetime [3-4], the physics of computation [5-7], and holography [8-12]. The holographic principle associates each bit within a spacelike three volume with a two-dimensional Planck-scale area on the surface of that volume. By

contrast, the quantum geometric limit associates each event or ‘op’ that occurs within a spacetime four volume with a two dimensional Planck-scale area on world sheets surrounding the volume. The quantum geometric limit follows from quantum physics on curved spacetime governed by Einstein’s equations. I also show that the derivation can be performed the other way around: Einstein’s equations follow from the assumption that each quantum event *removes* a two-dimensional Planck-scale area from spacetime.

Attempts to derive quantum limits to the accuracy of measuring the geometry of spacetime date back at least to the 1950s [3-4]. In discussions of the limits to measurement, care must be taken in applying arguments that rely on the Heisenberg uncertainty principle. In particular, the Heisenberg-based ‘standard quantum limit’ may be both standard and quantum, but it is not a fundamental limit to measurement: it can be surpassed using squeezed states and entanglement [1]. Fortunately, techniques from the physics of computation such as the Margolus-Levitin theorem [5], the bounds on the number of elementary logical operations and quantum bits allowed in a volume of spacetime [6], and the quantum speed limit [7], can be used to derive limits to the accuracy with which quantum systems can be used to measure spacetime geometry.

## 1 Limits to measuring spacetime

The Margolus-Levitin theorem [5] states that the time  $\Delta t$  it takes a quantum system such as a clock to go from one state to an orthogonal state is greater than or equal to  $\pi\hbar/2E$ , where  $E$  is the expectation value of the energy of the system above the ground state energy. The Margolus-Levitin theorem and its variants such as the quantum speed limit [7] can be combined with the fundamental physical limits to computation and measurement [1-2, 5-7] to put bounds on the accuracy to which spacetime geometry can be measured.

Consider Einstein’s seminal thought experiment for measuring spacetime geometry, in

which spacetime is filled with a ‘swarm’ of clocks that map out the spacetime geometry by exchanging signals with the other clocks and measuring the signals’ times of arrival (Figure 1). This thought experiment is of course the basis for the global positioning system (GPS). The clocks could be as large as GPS satellites, or as small as elementary particles. References [1-2] put bounds on how accurately this swarm of clocks can map out a sub-volume of spacetime with radius  $r$  over time  $t$ . Every tick of a clock or click of a detector is an elementary event in which a system goes from a state to orthogonal state. Accordingly, the the total number of ticks and clicks that can take place within the volume is limited by the Margolus-Levitin theorem: it is less than or equal to  $\# \equiv 2Et/\pi\hbar$ , where  $E$  is the quantum expectation value of the energy of the clocks within the volume, measured from their ground state. We call  $\#$  the number of quantum ‘ops’ accumulated by the system. A quantum op occurs when the state of the system accrues an average phase  $Et = \pi/2$  relative to the phase of the ground state, whether the system moves to an orthogonal state or not. For example, a thermal state of the system can be stationary but still accrue quantum ops.

If the clocks are packed too densely within some sub-volume, they will form a black hole and be useless for participating in the measurement of spacetime outside their horizon. To prevent black hole formation, the total energy of clocks and signals within spacelike regions of radius  $r$  must be less than  $c^4 r/2G$ . The Margolus-Levitin theorem together with the requirement that the clocks and signals within the volume be below black-hole density imply that the total number of elementary events and the number of ops that can occur in such a volume of spacetime are bounded by

$$\# \leq \frac{c^4 r t}{\pi \hbar G} = \frac{r t}{\pi l_P t_P}. \quad (1)$$

The usual Planck-scale limit arises from applying equation (1) to to the case of a single

clock performing a single tick. The quantum geometric limit (1) was derived without any recourse to quantum gravity: the Planck scale makes its appearance simply from combining quantum limits to measurement with the requirement that sub-volumes of our GPS system not form black holes.

Within a black hole, of course, arbitrarily large matter densities can occur, and so equation (1) might be violated in the approach to the singularity. Even if some sub-volume of our GPS does collapse into a black hole of Schwarzschild radius  $r$ , however, then equation (1) still holds according to an inertial observer far from the hole, with the number of elementary events corresponding to transitions in the Hawking radiation.

The holographic principle [8-12] encourages us to imagine each bit within a spatial three volume as projected onto the two-dimensional spacelike surface of the volume, at a density no greater than the Planck length squared. The quantum geometric limit (1) also associates an elementary event or quantum op with a two-dimensional Planck-scale area. It encourages us to imagine all the events within a four volume of spacetime as projected onto one-plus-one dimensional world sheets on the surface of that volume at a density of no greater than one event per Planck length times Planck time (Figure 2). That is, the quantum geometric limit represents a kind of ‘spacetime’ holography, in which ops rather than bits are projected onto two-dimensional surfaces.

Although straightforward to derive, the quantum geometric limit’s association of events with two-dimensional Planck scale areas is somewhat surprising. *A priori*, one might have thought that the number of events within a four volume would be limited by the measure of that volume divided by the Planck scale to the fourth power. Alternatively, holography could be taken to suggest that the number of events be proportional to the surface area of the volume times time divided by Planck scale cubed. The quantum geometric limit (1) shows that the concentration of possible events in spacetime is sparser

than either of these guesses indicate.

## 2 Covariant formulation

### 2.1 Covariant volumes

The derivation of the quantum geometric limit (1) given above is not covariant. Now rederive this limit in a covariant way. First, construct a covariant version of the notion of a four volume with temporal extent  $t$  and spatial radius  $r$ . To give a covariant version of the temporal extent of a four volume, consider a time-like curve (e.g., a geodesic) with length  $t$ . A covariant cylinder of radius  $r$  can then be constructed around this curve in a covariant fashion as follows (Figure 3).

Consider a segment of a time-like curve  $\Gamma$  with proper time length  $t$ , and let  $\tau$  be an affine parameter measuring proper time along the curve.  $\tau$  is the time ticked out by a clock following the curve. This curve can be used to define covariant spheres, cylinders, and solids of rotation using Einstein's technique for clock synchronization. For example, the sphere  $s(x, \tau)$  of radius  $x$  at time  $\tau$  is given by the intersection of the forward light cone from the point on the geodesic  $\tau - x/2c$  with the backward light cone from the point at  $\tau + x/2c$ . This construction works as long as the spacetime is not too irregular in the vicinity of  $\Gamma$ , e.g., the light cones do not cross an horizon on their way to the intersection point [11]. For  $\Gamma$  outside of any horizon, and for sufficiently small  $\tau, x$ , such covariant spheres can always be defined.

This construction is clearly covariant, and allows us to define spheres at any point along the curve up to the radius at which horizons and caustics intervene (e.g., if the clock is uniformly accelerated, the distance to the Rindler horizon [13-14]). The same construction allows us to define a covariant cylinder of radius  $r$  and length  $t$  centered on  $\Gamma$  as the collection of spheres  $s(x, \tau)$  of radius  $x \leq r$  at times  $0 \leq \tau \leq t$ . Arbitrary solids

of rotation of varying radius can similarly be defined by letting the radius of the spheres vary with  $\tau$ . A covariant version of the notion of ‘a volume of spacetime of radius  $r$  over time  $t$ ’ in the quantum geometric limit (1) then becomes ‘a volume of spacetime contained within the covariant cylinder of radius  $r$  centered on a timelike geodesic of proper length  $t$ .’

## 2.2 Covariant limit to the number of events

Next construct a covariant version of the energy-time limit for the number of events. The number of times a clock ticks or a detector clicks is a scalar quantity, agreed upon by all observers. That is, although they may express the quantum operators corresponding to when a clock ticks or detector clicks in different forms, all observers agree whether or not an internal clock or detector variable has moved from one state to an orthogonal state within a particular spacetime volume. By contrast, the question whether a vectorial quantity such as the average position  $\bar{x}_\mu$  of a clock or particle has changed state has an observer-dependent answer. For example, an observer for whom a particle is at rest will regard that particle as remaining in the same position state, while a moving observer will regard the particle as having moved to an orthogonal position state.

To make the full quantum geometric limit (1) covariant, imagine that a four volume of spacetime  $\mathcal{V}$  is filled with a swarm of massive clocks [14] whose paths make up a congruence of time-like curves with time-like unit tangent vector  $V^\mu$ ,  $g_{\mu\nu}V^\mu V^\nu = -1$ . The local energy density as measured by a clock with this tangent vector is  $T_{\mu\nu}V^\mu V^\nu$ , where  $T_{\mu\nu}$  is the energy momentum tensor, including clocks and signals. To apply the Margolus-Levitin theorem, the ground state energy is taken to be zero, so that this energy density is non-negative (the weak energy condition [14]). According to this set of clocks, the maximum number of events in the four volume including ticks of clocks, clicks of

detectors, and motion of signals, is then limited by the number of quantum ops,

$$\# = (2/\pi\hbar) \int_{\mathcal{V}} T_{\mu\nu} V^\mu V^\nu \sqrt{-g} dv. \quad (2)$$

The covariant version of the quantum geometric limit for number of the number of ops within a covariant four cylinder of radius  $r$  and temporal extent  $t$  is obtained by requiring that the effective mass within that cylinder be no greater than the mass of a black hole of radius  $r$ . To obtain this effective mass, take  $V^\mu$  to define the instantaneous ‘rest frame’ of the matter (the frame in which  $T_{\mu\nu}$  is diagonal). For example, if we model the clocks as a perfect fluid, then the  $V^\mu$  are the tangents to the flow lines of the fluid. Demanding that the number of ops  $\#$  defined in (2) to be no greater than  $rt/\pi\ell_P t_P$  gives the covariant version of the quantum geometric limit.

### 3 Deriving Einstein’s equations from the quantum geometric limit

The holographic principle arose out of black-hole thermodynamics and quantum field theory on curved spacetime. In [15], Jacobson turned the argument around: he showed how Einstein’s equations can be derived from combining the holographic entropy-area law with the fact that accelerated observers see horizon radiation with a temperature proportional to their acceleration. (See also the work of Verlinde [16] and Dreyer [17].)

The quantum geometric limit arises from applying the physics of computation to the concept that black holes are maximally dense objects. The limit implies that each quantum event in spacetime is associated with a two-dimensional Planck-scale area. Now turn the argument around and derive Einstein’s equations from the quantum geometric limit. The basic idea is straightforward: each elementary quantum ‘op’ *removes* a Planck scale area from a one-plus-one dimensional space-time surface on the boundary of the

volume, or equivalently, from the two-dimensional sections of the spatial three volume in which the event occurs (Figure 4). The removal of area from a flat two-dimensional section causes it to curve (Figure 5). This curvature is the curvature of spacetime, and as will now be shown, it induces the spacetime to obey Einstein's equations.

Consider an observer following a path through spacetime. The observer describes her local region of spacetime by an orthonormal tetrad of vectors (a vierbein)  $\{e^a_\mu\}$ .  $e^0_\mu$  is the timelike tangent vector to her path, and  $e^j_\mu$ ,  $j = 1, 2, 3$  are an orthonormal triad of spacelike vectors. We have  $e^a_\mu e^b_\nu g^{\mu\nu} = \eta^{ab}$ , where  $\eta^{ab} = \text{diag}(-1, +1, +1, +1)$  is the Minkowski-space metric, and  $g^{\mu\nu}$  is the spacetime metric. Similarly,  $e^a_\mu e^b_\nu \eta_{ab} = g_{\mu\nu}$ . Latin indices of  $e^a_\mu$  are raised and lowered using  $\eta_{ab}$ , and Greek indices are raised and lowered using  $g_{\mu\nu}$ . Consider a covariant cylinder of radius  $r$  and temporal extent  $t$  centered on the observer with the temporal axis of the cylinder oriented in the  $e^0_\mu$  direction. The average energy density above the ground state energy within this cylinder as measured by the observer is  $T^{\mu\nu} e^0_\mu e^0_\nu$ . Take  $r, t$  to be sufficiently small that the energy density and the curvature tensor are essentially constant throughout the cylinder.

A quantum 'op' corresponds to an average accumulation of phase  $E\Delta t = \pi/2$  in the observer's frame, where  $E$  is the observed energy above the local ground state energy. When a clock ticks or a detector clicks, at least one quantum op is performed. The total number of ops recorded by the observer is

$$\# = (2/\pi\hbar) T^{\mu\nu} e^0_\mu e^0_\nu (4\pi/3) r^3 t. \quad (5)$$

The quantum geometric limit suggests that we imagine each op that takes place within the four cylinder to be projected onto Planck-scale area  $A = \alpha \ell_P t_P$  located on suitable one-plus-one dimensional world sheets on the surface of the cylinder (Figure 2). Let  $C_{ij}$  be the circle of radius  $r$  in the  $ij$  plane,  $i, j = 1, 2, 3$ . Let  $W_{ij}$  be the one-plus-one dimensional



cylindrical world sheet traced out by the circle  $C_{ij}$  over time  $t$ . The three world sheets  $W_{ij}$  effectively ‘surround’ the covariant four cylinder, and so are suitable world sheets on which to project the ops that occur within that cylinder. Now assume that each op that occurs within the four cylinder removes area  $\alpha \ell_P t_P$  from the world-sheets  $W_{ij}$ . As shown in Figure 4, the removal of area decreases both the circumferences of the circles  $C_{ij}$ , and the areas  $A_{ij}$  of the spacelike disks bounded by  $C_{ij}$ , when compared with their values in flat space. As a result, spacetime curves (Figure 5). In fact, as will now be shown, it curves in exactly such a way that Einstein’s equations hold.

First, note that the area removed from the world sheets determines the Gaussian curvatures of the circular spacelike cross-sections of the cylinder. In the limit of small  $r$  the Bertrand-Diquet-Puiseux theorem implies that the Gaussian curvature  $K_{ij}$  of the  $ij$  cross-section of the cylinder is related to the area  $\Delta A_{ij}$  removed by  $\Delta A_{ij} = \pi r^4 K_{ij}/12$ . Equivalently,  $\Delta c_{ij} = \pi K_{ij} r^3/3$ , where  $\Delta c_{ij}$  is the magnitude of the decrease in the circumference of the disk. In the limit of small  $t$ , the area removed from the world sheet  $W_{ij}$  is related to the Gaussian curvature of the disk by  $t \Delta c_{ij} = \pi K_{ij} r^3 t/3$ .

Next, recall that the Gaussian curvature of a two-dimensional cross-section equal to its sectional curvature [18]:

$$K_{ab} = \frac{R^\mu{}_{\nu\rho\sigma} e_a{}^\rho e_b{}^\sigma e_b{}^\nu e_{a\mu}}{e_a{}^\rho e_{a\rho} e_b{}^\sigma e_{b\sigma} - (e_a{}^\rho e_{b\rho})^2} = \pm R^\mu{}_{\nu\rho\sigma} e_a{}^\rho e_b{}^\sigma e_b{}^\nu e_{a\mu}. \quad (6)$$

where  $R^\mu{}_{\nu\rho\sigma}$  is the Riemann tensor. The  $+$  in the  $\pm$  holds when  $a, b$  are both spacelike indices, and the  $-$  holds when one is timelike, e.g.,  $a = 0$ . Accordingly, the Ricci tensor evaluated along one of the tetrad vectors is related to the sum of the sectional curvatures along the three sections in which that vector participates: e.g.,  $R^{\mu\nu} e^0{}_\mu e^0{}_\nu = -K_{01} - K_{02} - K_{03}$ , and  $R^{\mu\nu} e^1{}_\mu e^1{}_\nu = K_{01} + K_{12} + K_{13}$ .

Removing a total area  $\alpha \ell_P t_P \#$  from the three world sheets  $W_{ij}$  surrounding the cylin-

der then yields

$$\begin{aligned}
\alpha \ell_P t_P \# &= (\pi/3)(K_{12} + K_{23} + K_{31})r^3 t \\
&= (\pi/6)(R^{\mu\nu} e^0{}_\mu e^0{}_\nu + \sum_{j=1}^3 R^{\mu\nu} e^j{}_\mu e^j{}_\nu) r^3 t \\
&= (\pi/3)(R^{\mu\nu} - (1/2)g^{\mu\nu} R) e^0{}_\mu e^0{}_\nu r^3 t. \tag{7}
\end{aligned}$$

Combining equation (7) for the change in area together with equation (5) for the total number of ops yields

$$(8/\pi\hbar)\alpha\ell_P t_P T^{\mu\nu} e^0{}_\mu e^0{}_\nu = (R^{\mu\nu} - (1/2)g^{\mu\nu} R) e^0{}_\mu e^0{}_\nu. \tag{8}$$

Setting  $\alpha = \pi^2$ , and noting that  $e^0{}_\mu$  can be any timelike unit vector – that is, equation (8) should hold for all observers – implies Einstein’s equations.

## 4 Vacuum energy and entanglement

Equations (7-8) show that if each op removes an area  $A = \pi^2 \ell_P t_P$  from the one-plus-one dimensional world sheets surrounding the spacetime volume in which the op takes place, inducing curvature, then the resulting curved spacetime obeys Einstein’s equations without an intrinsic cosmological constant. The lack of an intrinsic cosmological term arises from the requirement – enjoined by the quantum mechanics of measurement – that the number of events or ops be an observable quantity. Consequently, that number can depend only on the energy above the ground state energy. If only observable quantum phases contribute to the interaction between matter and geometry, then the vacuum energy does not contribute to the gravitational energy. Of course, the equations do not rule out a form of matter that corresponds to a cosmological term.

Quantum field theory shows that in the presence of mass and charge, that vacuum possesses a rich structure (vacuum polarization, etc.). In the presence of curvature, this

vacuum structure can give rise to significant effects such particle creation [13]. A quantum field theoretic treatment lies outside of the scope of the current paper, however, and will be taken elsewhere.

In general, the quantum state of the local matter is a mixture, e.g., a thermal state. Even when the overall state of the matter is pure, the local state is typically mixed due to entanglement: for example, vacuum entanglement is responsible for the entropy-area law [19]. The Page-Geilker experiment [20] suggests that – at least in relatively macroscopic situations – different components of the mixture yield different local geometries. For a mixed state, the number of ops can be evaluated separately in each component of the mixture. In an entangled state, the geometry induced by a particular component of the local mixed state is correlated with the geometry induced by the corresponding state of the matter elsewhere. Again, further work is required to elucidate the role of entanglement, including the connection between Jacobson’s work [15] the derivation of Einstein’s equations given here.

## 5 Conclusion

Intriguing connections between quantum information and gravity have been arising for decades [1-4, 6, 8-12, 15-22]. This paper attempted to elucidate those connections by applying fundamental quantum limits to measurement of space and time. As in [15], the result is not a theory of quantum gravity *per se*, but rather a quantum theory which gives rise to general relativity under simple assumptions. The quantum geometric limit states that the number of elementary events such as clock ticks, detector clicks, or bit flips that can be contained in a four volume of space time of covariant radius  $r$  and spatial extent  $t$  is limited by  $rt/\pi l_P t_P$ . By bounding the number of ops that can take place within a four volume, this limit is complementary to the holographic limit, which bounds the number

of bits associated with a spatial three volume by the surface area of the volume divided by the Planck length squared. Holography encourages us to imagine the bits of information characterizing the quantum state of the spatial three volume as projected onto the two-dimensional boundary of the volume at a density no greater than on the order of one bit per Planck length squared. The quantum geometric limit encourages us to imagine the elementary events or ‘ops’ that occur within a spacetime four volume as projected onto 1+1 dimensional surfaces on the boundary of that volume at a density no greater than on the order of one op per Planck length per Planck time. If each op removes an area  $\pi^2 \ell_P t_P$  from these surfaces, then Einstein’s equations hold.

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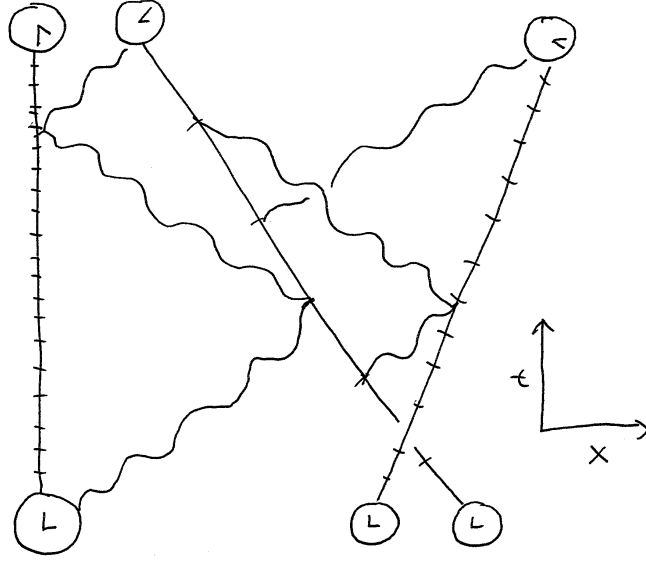


Figure 1: Clocks in space: The GPS system consists of clocks falling inertially through space, sending each other signals at the speed of light. Proposed by Einstein as a *gedanken-experiment*, GPS measures the geometry of spacetime by recording when signals depart from one clock and when they arrive at another. Demanding that clocks and signals within a volume not exceed the black-hole density leads to the quantum geometric limit: the total number of ticks of clocks and clicks of detectors that can occur within a volume of space-time of radius  $r$  and over time  $t$  is bounded by  $\# = rt/\pi\ell_P t_P$ , where  $\ell_P$  is the Planck length and  $t_P$  is the Planck time.

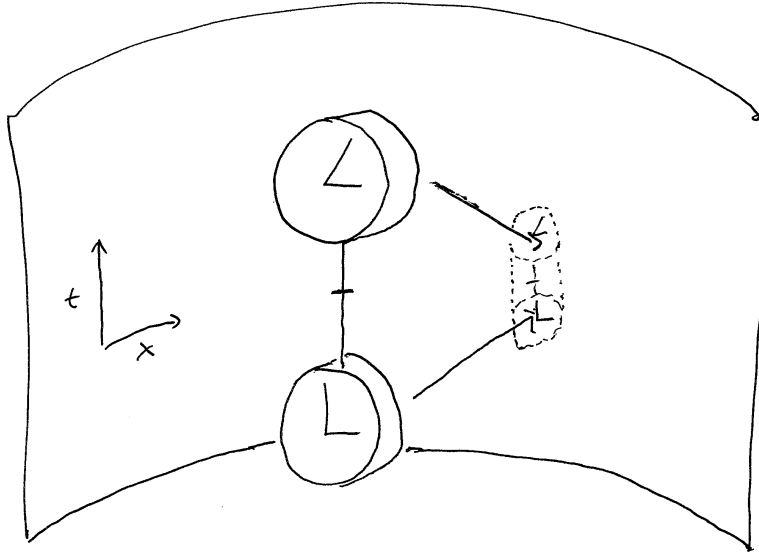


Figure 2: The quantum geometric limit as space-time holography: The quantum geometric limit encourages us to imagine events such as the tick of a clock that occur within a covariant cylinder as projected onto a 1+1 Planck-scale area on the surface of the cylinder. The projected event could be localized to a particular Planck-scale area, as shown here, or delocalized holographically over the surface of the cylinder.



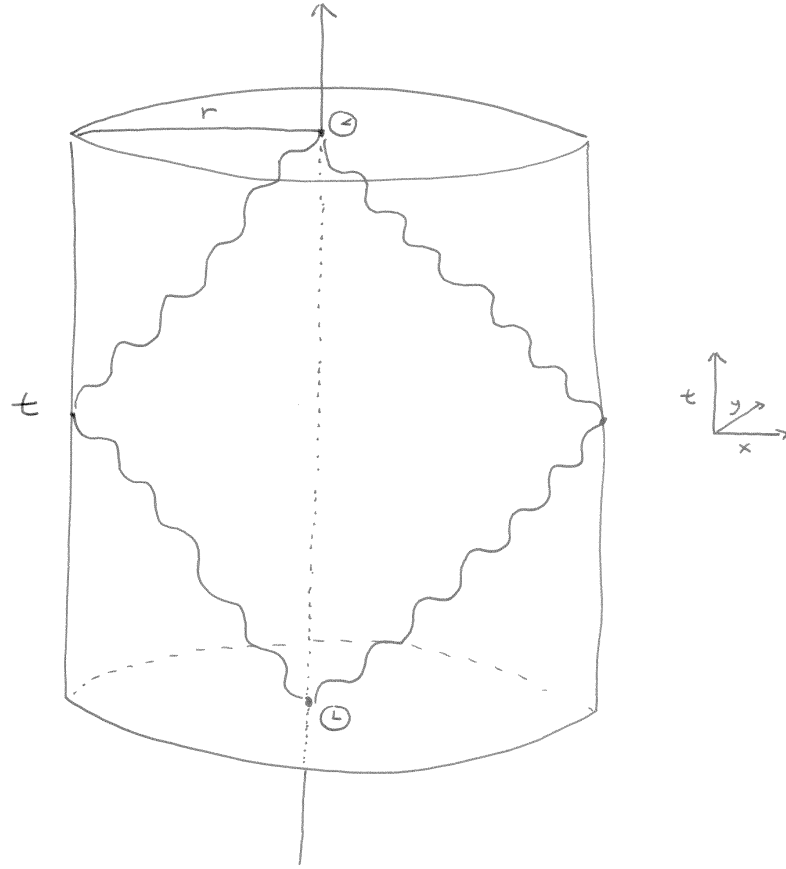


Figure 3: A covariant cylinder: A covariant cylinder is the space-time volume traced out by a sphere of radius  $r$  over time  $t$ . A GPS clock maps out a covariant cylinder by identifying points from which light signals take time  $r/c$  to propagate to and from the clock.

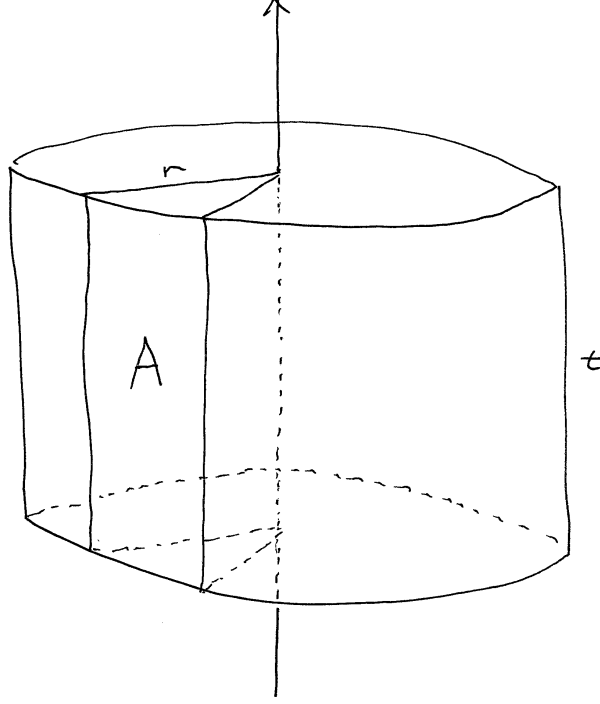


Figure 4: Ops remove area from two-dimensional surfaces: The figure shows a 2+1 dimensional cross section of a 3+1 dimensional covariant cylinder. The size of the cylinder is taken to be sufficiently small that the energy density is essentially constant throughout the cylinder. Each op that takes place within the cylinder removes a Planck-scale area from the 1+1 dimensional space-time boundary of the cross-section. The total area removed is  $A = \pi^2 \ell_P t_P \#$ , where  $\#$  is the number of ops.

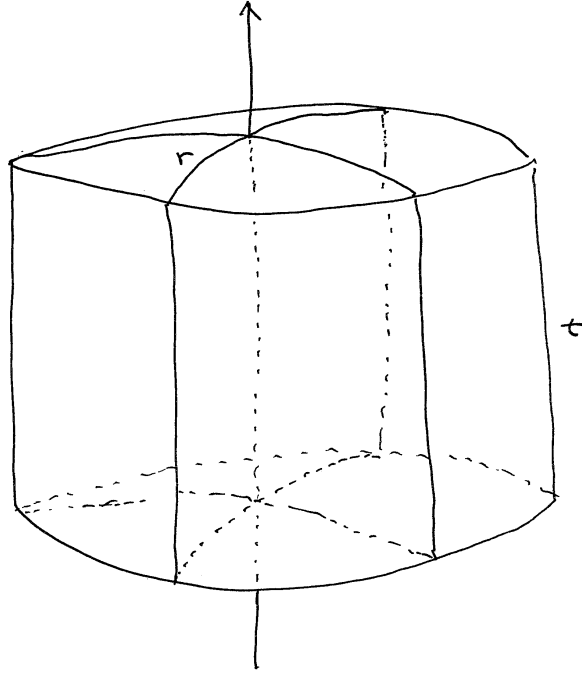


Figure 5: As a result, spacetime curves: Removal of area  $A$  from the 1+1 dimensional boundary of the cylinder also removes area from the 2 dimensional spacelike cross sections of the cylinder. This removal of area causes those sections to curve. The Gaussian curvature of the section is  $K = 3A/\pi r^3 t$ . This sectional curvature in turn causes the spacetime to obey Einstein's equations.